Some Results of Suboptimal Gust Alleviation

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Nomenclature

a_{cz} = linear acceleration of aircraft center of mass in z direction, fps^2
C_{L0} = lift coefficient at trim angle of attack
g = local acceleration due to gravity, fps^2
h = aircraft altitude, ft
$K_1 \to K_5$ = control system gains
L_w = gust characteristic length, ft
n = $-a_{cz}/g$, normal acceleration factor
$\hat{q}$ = aircraft pitch rate,
U_0 = aircraft forward velocity, fps
w_g = gust vertical velocity, fps
xyz = aircraft stability axis system
α_g = $-w_g/U_0$, angle-of-attack perturbation due to gust, measured at aircraft center of mass, rad
η = elevator angle, measured from trim, rad
Ω = spatial frequency, rad/ft
σ_w = rms vertical gust velocity, fps

Introduction

IN Ref. 1, the results of a simplified gust alleviation design were discussed. The problem of minimizing the mean square normal acceleration of the center of gravity of a large, rigid, jet transport flying level through a one-dimensional, turbulent upwash field was considered. A stationary, homogeneous, isotropic, frozen turbulence field was assumed and a simplified power spectrum utilized:

$$\Phi_{w_g}(\Omega) = 2\sigma_w{}^2 L_w/[1 + (L_w\Omega)^2]$$

The index of performance was given by

$$J = \lim_{T\to\infty} \frac{1}{T}\int_0^T [qn^2(t) + ru^2(t)]dt = \overline{qn^2} + \overline{ru^2}$$

where $n(t)$ represented the normal acceleration factor and $u(t)$ the signal driving the elevator-servo combination.

Two flight conditions were studied: a 30,000 ft, 500 mph cruise condition and a sea level, 200 mph landing approach. The characteristic length L_w of the gust spectral model was varied from 500 to 6000 ft.

Figure 1 is a block diagram of the gust alleviation system obtained by applying the methods of linear optimal control. Gains K_1–K_3 were functions of the flight condition alone, i.e. altitude h, forward velocity U_0 and trim lift coefficient C_{L0}. Gain K_4 depended upon both flight condition and gust characteristic length. It was shown that considering K_4 to be a function of flight condition alone did not result in serious performance decrements for the range of characteristic lengths studied.

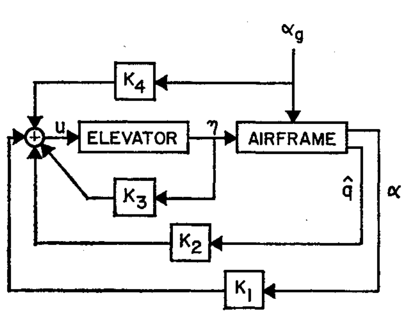

Fig. 1 Block diagram for optimal gust alleviation system.

It was suggested that α_g, the vertical gust angle-of-attack perturbation, could be obtained in an indirect fashion by measuring the normal acceleration factor n in addition to α, $\hat{q}$, and η. There it was stated that n could be written

$$n = C \cdot [-C_{z_\alpha}\alpha - C_{z_\eta}\eta - C_{z_\alpha}\alpha_g]$$

where the constant C was a known function of the flight condition. Rearranging

$$\alpha_g = -n/CC_{z_\alpha} - \alpha - C_{z_\eta}\eta/C_{z_\alpha}$$

Thus, measurement of n, α, and η would yield information about α_g. This indirect measurement scheme was necessary since direct determination of α_g using aerodynamic devices such as incidence vanes or pressure probes appeared fraught with problems.

This Note summarizes some new results regarding the gust alleviation design, wherein the desirability of on-line information about α_g is demonstrated from the standpoint of mean square performance. In addition, the feedback variables are restricted to those which can be measured most readily. Finally, the resulting feedback configuration is shown to be similar in form to a simple "acceleration autopilot" and to utilize what have been termed "essential feedbacks" normally associated with effective vertical gust alleviation.

Parameter Optimization

To determine the importance of on-line information about α_g, a brief parameter optimization study was performed on a suboptimal configuration identical in form to that of Fig. 1 except with gust information deleted, i.e. $K_4 = 0$. Values of K_1–K_3 which minimized J for this configuration were obtained via a direct search, digital technique.[2] The resulting mean-square normal acceleration factors were found to be an average 20% larger than those of the optimal system for the two flight conditions mentioned. In addition, the gains K_1–K_3 had to be considered functions of both flight condition and gust characteristic length. Finally, transient performance could no longer be manipulated via the root square locus technique as was the case with the optimal design.

Measurable Variables

The optimal system of Ref. 1 assumed exact, instantaneous knowledge of α, $\hat{q}$, η, and n. Of these four variables, angle-of-attack α represents the most formidable measurement problem. The necessity of measuring angle of attack can be side-stepped as follows. For the aircraft studied, $C_{z_\eta} \ll C_{z_\alpha}$, and the normal acceleration factor can be expressed in an approximate but accurate fashion as

$$n = -CC_{z_\alpha} \cdot [\alpha + \alpha_g]$$

Figure 1 indicates that if K_1 and K_4 were equal, measurement of α would be unnecessary. Indeed, only $\alpha + \alpha_g$ would be required and this sum could be obtained from measurement of n as indicated above. Considering gain K_4 to be only a function of flight condition and equating it to K_1 results in an average mean square normal acceleration increase of only 2% over the optimal system in which K_4 was a function of both flight condition and gust characteristic length. It should be emphasized that manipulating K_4 has no effect upon the transient performance which was determined once optimal values of K_1–K_3 were found.[1] The final alleviation system is shown in Fig. 2.

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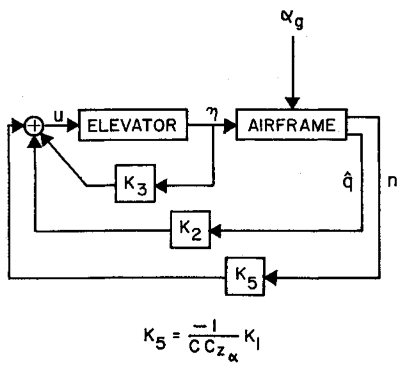

Fig. 2 Block diagram for alleviation system using essential feedbacks.

Summary

At this point it is interesting to summarize the analysis which has led to the configuration of Fig. 2. Linear optimal control theory was applied to the aircraft and gust mathematical models as outlined previously. The root square locus technique was utilized to select q/r, the ratio of performance index weighting factors. This choice was aimed at ensuring acceptable transient performance. The optimal gains K_1–K_3 were found to be functions of flight condition alone. The optimal gain K_4 was found to be a function of both flight condition and gust characteristic length. Ignoring the latter dependence produced no significant performance penalties. Indirect measurement of α_g was proposed and finally the feedbacks were restricted to easily measurable quantities. The resulting simple configuration exhibits nearly optimal mean square performance, gains which are independent of gust characteristic length and transient behavior which is superior to the open-loop system.

Conclusions

The following can be added to the conclusions of Ref. 1. For the particular aircraft and flight conditions studied: 1) The alleviation system which utilized gust perturbation information offered considerably better mean square performance than the suboptimal configuration which did not. 2) Indirect, nonaerodynamic measurement of gust angle-of-attack perturbation α_g appears feasible through measurement of the normal acceleration factor n. 3) For the range of gust characteristic lengths considered, restricting the feedback variables to those which can be readily measured results in only an average 2% mean square normal acceleration increase over the optimal system. 4) The final alleviation system shown in Fig. 2 is similar in form to a simple "acceleration autopilot"[3] and utilizes "essential feedbacks" normally associated with effective vertical gust alleviation.[4]

References

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